28. M. D. Lilley, unpublished data.
32. GenBank accession number AF393379.
33. GenBank accession number AF393378.
34. GenBank accession number AF393377.
36. GenBank accession number AF393376.
37. Scaly-foot refers to the presence of scales covering the sides of the foot. The scales appear to be homologous to the operculum of other gastropods.
38. Epi- and endosymbioses were confirmed with the use of 4,6-diamidino-2-phenyldine (DAPI) staining and epifluorescence microscopy, transmission electron microscopy (TEM), and tests for the CO2-fixing enzyme of the Calvin cycle, Rubisco, including activity and immunoblot assays after methods described in [39, 41, 56].
40. Stable isotope analyses of acid-furred tissues were made by R. Petty, Analytical Laboratory, Marine Science Institute, University of California, Santa Barbara, CA. Carbon stable isotope values are reported in δ notation relative to the Pee Dee Belemnite standard: δ13C = ([(13Csample/12Csample)/(13Cstandard/12Cstandard)] - 1) × 1000; δ15N = 1000 × ([15Nsample/14Nsample]/[15Nstandard/14Nstandard] - 1).
42. Biological fractionation of carbon isotopes is not fully understood where CO2 is limiting, which results in carbon isotope values close to or identical to that of the source dissolved inorganic carbon (−6 to −8 %) [57].
44. In community-level analysis of nitrogen isotope compositions of vent animals, the most 15N-depleted values are typically associated with symbiont-bearing invertebrates.
45. Taxonomic specialists consulted include D. Fautin (Cnidaria); S. Kojima and T. Okutani, A. Scheltema, and A. Wärn (Mollusca); J. Blake, Desbruyères, S. Herzig, and T. T. Polyxenia; W. Newman and T. Yamaguchi (Cirripedia); and S. Tsuchida and H. Watabe (Crustacea).
47. C. L. Van Dover, Mar. Ecol. Prog. Ser., in press.
48. Novel sequences obtained from Kairei animal species were deposited with GenBank under accession numbers: AF004627–AF004629, AF399955–AF399964, and AF401175. DNA was extracted using the DNeasy Tissue Kit (Qiagen, Valencia, CA). Mitochondrial DNA (mtDNA) was amplified with the use of ND4 primers (Arghyl and NAFV) for mussels (58–59), universal COI primers (HO2-2198 and LCO-1490) for common mussels and chimaeras and shrimp [60], universal cytosome b (Cytb) primers (UCytb144F and UCytb270R) for crabs (61), and T6S DNA primers (16Sar and 16Sbr) for hairy gastropods and limpets [62]. The gene of choice for a particular organism was based on published or ongoing molecular systematic studies of these taxa. Polymerase chain reaction (PCR) products were sequenced directly using ABI 377 or Llicor 4000L sequencers (Lincoln, NE). In all cases, both forward and reverse strands were sequenced aliquoting Kairei mussels (N = 6) and shrimp (N = 6) were compared with mussel and shrimp sequences from Pacific and Atlantic localities (7, 49; Y. Won, unpublished data). The communal polychaete (N = 6) was compared with individuals from Pacific and Atlantic localities (50; S. Houde, P. Chevaldorne, unpublished data). Bythograeid crabs (N = 9) were compared with individuals from Pacific and Atlantic localities (L. Hurtado, unpublished data). Hairy gastropods (N = 2) were compared with individuals from the Mariana Trough (N = 5) (S. Goffredi, unpublished data). Lepetodrilus limpets (N = 2) were compared with specimens from other Pacific localities.
51. Edmond mussels (N = 6) were compared with mussel sequences from Kairei individuals (N = 6; Y. Won, unpublished data). A single bythograeid crab from Edmond was compared with individual Micromao Kairei (N = 9; L. Hurtado, unpublished data). Edmond Alviniconcha snails (N = 2) were compared with individuals from Kairei (N = 2; S. Goffredi, unpublished data).
52. At present, we know neither the spacing or biological character of hydrothermal systems along the southern Mid-Atlantic Ridge due to lack of adequate exploration and mapping.
65. We thank the Captain and crew of the R/V Knorr and RV Jason for their invaluable assistance at sea. Shore-based and shipboard technical and engineering staff of the WHOI Deep Submergence Group played a key role in the acquisition of data. L. Dolby, R. Kunig, and J. Philley provided scientific support at sea. S. Houde and P. Chevaldorne provided DNA sequence data for Branchiopodan petioninaeae. J. Thomas assisted with TEM studies; C. Jenkins and J. Bonaventura reviewed drafts of the manuscript. NSF Ocean Sciences Divisions of Biological Oceanography and Geology and Geophysics (OCE9712358 to CLVD and OCE9910799 to RCV) supported the U.S. field program.
66. 19 July 2001; accepted 31 August 2001
67. Published online 13 September 2001; 10.1126/science.1064574
68. Include this information when citing this paper.

A Four-Dimensional Generalization of the Quantum Hall Effect
Shou-Cheng Zhang and Jianguo Hu

We construct a generalization of the quantum Hall effect, where particles move in four dimensional space under a SU(2) gauge field. This system has a macroscopic number of degenerate single particle states. At appropriate integer or fractional filling fractions the system forms an incompressible quantum liquid. Gapped elementary excitation in the bulk interior and gapless elementary excitations at the boundary are investigated.

Most strongly correlated systems develop long-range order in the ground state. Familiar ordered states include superfluidity, superconductivity, antiferromagnetism, and charge density wave (J). However, there are special quantum disordered ground states with fractionalized elementary excitations. In one-dimensional (1D) systems, Bethe’s Ansatz (2) gives exact ground-state wave functions of a class of Hamiltonians, and the elementary excitations are fractionalized objects called spinons and holons. In the 2D quantum Hall effect (QHE) (3, 4), Laughlin’s wave function (J) describes an incompressible quantum fluid with fractionally charged elementary excitations. This incompressible liquid can also be described by a Chern-Simons-Landau-Ginzburg field theory (5), whose long-distance limit depends only on the topology but not on the metric of the underlying space (6). These two special quantum disordered ground states are the focus of much theoretical and experimental studies, because they give deep insights into the interplay between quantum correlations and dimensionality and into how this interplay can give rise to fractionalized elementary excitations.

In view of their importance, it is certainly desirable to generalize these quantum wave functions to higher dimensions. However, despite repeated efforts, the Bethe’s Ansatz solutions have not yet been generalized to dimensions higher than one. Laughlin’s wave function uses properties that seem to be special to the 2D space. In this work, we shall report the generalization of the quantum Hall system to four space dimensions, and this system shares many compelling similarities with the 2D counterparts. In the 2D QHE, the charge current is carried in a direction perpendicular to the applied electric field (and also perpendicular to the magnetic field, which...
Here \( m = -1, -1 + 1, \ldots, I - 1, I \), therefore the ground state is \( 2I + 1 \) fold degenerate. Any states in the III can be expanded in terms of a homogeneous polynomial of \( \phi_1 \) and \( \phi_2 \) with degree \( 2I \). Notice that the conjugate coordinate \( \phi^*_n \) does not enter the wave function in the III.

We see that the crucial algebraic structure of the QHE problem is the fractionalization of a vector coordinate into two spinor coordinates. Therefore, in seeking a higher dimensional generalization of the QHE problem, we need to find a proper generalization of Eq. 1. As the generalization of the three Pauli matrices is the five \( 4 \times 4 \) Dirac matrices \( \Gamma_{ab} \) satisfying the Clifford algebra \( \{ \Gamma_{ab}, \Gamma_{cd} \} = 2 \delta_{ac} \delta_{bd} \), we generalize Eq. 1 to

\[
\begin{align*}
x_a &= \Psi_a (\Gamma_{a,b}) \Psi_b, \\
\Psi_a &= \gamma \end{align*}
\]

Here, \( \Psi_a \) is a four-component complex spinor with \( \alpha = 1, 2, 3, 4 \), and \( \gamma \) is a five-component real vector. From the normalization condition of the \( \Psi \) spinor it may be seen that \( \chi^2 = 1 \), therefore, \( \chi_R = R \chi \) describes a point of the 4D sphere \( S^4 \) with radius \( R \). From this heuristic reasoning, one may hope to find a 4D generalization of the QHE problem, where the wave functions in the ground states are described by the products of \( \Psi \) spinors, in a natural generalization of Eq. 2. Equations 1 and 3 are known in the mathematical literature as the first and the second Hopf maps (11). The problem now is to find a Hamiltonian for which these are the exact ground state wave functions.

An explicit solution to Eq. 3 can be expressed as

\[
\Gamma^{(1,2,3)} = \begin{pmatrix} 0 & -i \sigma_l \\ -i \sigma_l & 0 \end{pmatrix}, \quad \Gamma^4 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \Gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

(4)

\[
\begin{align*}
\Psi_1 &= \sqrt{1 + x_1 u_1} u_1 \\
\Psi_2 &= \sqrt{2(1 + x_1)} x_1 - i x_2 \sigma_l \\
\Psi_3 &= \sqrt{1 + x_2} u_2 \\
\Psi_4 &= \sqrt{2(1 + x_2)} x_2 - i x_3 \sigma_l
\end{align*}
\]

(5)

where \( u_1, u_2 \) is an arbitrary two-component complex spinor satisfying \( u_a u_a = 1 \). Any \( SU(2) \) rotation on \( u_a \) preserves the normalization condition and maps to the same point \( x_a \) on \( S^4 \). From the explicit form of \( \Psi_a \), one can compute the geometric connection (Berry’s phase) \( \Psi_a d \Psi_a^* \). Since the differentiation operator \( d \) acts on the vector coordinates \( x_a \), subject to the condition \( x_a d x_a = 0 \). One finds \( \Psi_a d \Psi_a^* = i \nu_a \alpha u_{a \nu} u_{a \nu}, \nu_a = 0 \), and

\[
\begin{align*}
\alpha_1 &= -i \\
\alpha_2 &= -i \\
\alpha_3 &= -i \\
\alpha_4 &= -i \\
\eta_1 &= -i \\
\eta_2 &= -i \\
\eta_3 &= -i \\
\eta_4 &= -i
\end{align*}
\]

(6)

where \( \nu_a \) is also known as the t’Hooft symbol. \( \eta_a \) is the \( SU(2) \) gauge potential of a Yang monopole defined on \( S^4 \) (12). Upon a conformal transformation from \( S^4 \) to the 4D Euclidean space \( R^4 \), this gauge potential is transformed to the instanton solution of the \( SU(2) \) Yang-Mills theory (14). We shall call \( I \) a \( SU(2) \) isospin matrix and the gauge potential defined in Eq 6 can be generalized to an arbitrary representation \( I \) of the \( SU(2) \) Lie algebra \( [I_a, I_b] = i \epsilon_{abc} I_c \). The gauge field strength can be calculated from the form of the gauge potential. From the covariant derivative \( D_i = \partial_i + a_i \), we define the field strength as \( F_{ab} = [D_a, D_b] \). Both \( a_i \) and \( F_{ab} \) are matrix valued and can be generally expressed in terms of the isospin components \( a_i \) and \( a_{ab} \), which could be used to model the relativistic fundamental lower limit on all length scales. This feature shares similarity to noncommutative quantum field theory and string theory of elementary particles.

A 4D generalization of the quantum Hall problem. In the QHE problem, it is advantageous to consider compact spherical spaces that can be mapped to the flat Euclidean spaces by standard stereographical mapping (10). Eigenstates in the QHE problem are called Landau levels, and we first review the lowest Landau level (III) defined on the 2D sphere, denoted by \( S^2 \). A point \( x_a \) on \( S^2 \) with radius \( R \) can be described by dimensionless vector coordinates \( x_i = x_i/R \) with \( i = 1, 2, 3 \), which satisfy \( x_i^2 = 1 \). However, \( S^2 \) has a special property that one can also take the “square root” of the vector coordinate \( x_i \) through the introduction of the complex spinor coordinates \( \phi_a \), with \( \alpha = 1, 2 \). These spinor coordinates are defined by

\[
x_a = \phi_1 \phi_2 = \phi_3 \phi_4 = 1
\]

where \( \phi_i \) are the three Pauli spin matrices. If there is a magnetic monopole of strength \( g \) at the center of \( S^2 \), satisfying the Dirac quantization condition \( eg \theta = I - \text{integer or half integer} \), then the normalized eigenfunctions in the III are just the algebraic products of the spinor coordinates

\[
\langle x | \phi_n \rangle = \sqrt{\frac{(2I)!}{(I!)(I - m)!}} \phi_1^{x_1} \phi_2^{x_2} \]

(2)
+ \frac{q^2}{2} + 2p + q \) and \( d(p, q) = (1 + q)(1 + p - q) \left( 1 + \frac{p + q}{3} \right) \) respectively.

However, for a given \( I \), these two integers are related by \( p = 2I + q \). One can show that \( \Sigma_{\alpha} A_{\alpha I} = \Sigma_{\alpha} C_{\alpha I} = 2I^2 \). Therefore, for a given \( I \), the energy eigenvalues of the Hamiltonian (Eq. 7) are given by

\[
E(p = 2I + q, q) = \frac{k^2}{2M R^2} C(p = 2I + q, q) = 2[I(I + 1)]
\]

with degeneracy \( d(p = 2I + q, q) \). The ground state, which is the lowest \( SO(5) \) level for a given \( I \), is obtained by setting \( q = 0 \), and we see that it is \( \frac{k}{p} (p + 1)(p + 2)(p + 3) \) fold degenerate. Therefore, the dimension of the \( \mathcal{S}(2) \) representation plays the role of the magnetic flux, while \( p \) plays the role of the Landau level index. States with \( q > 0 \) are separated from the ground state by a finite energy gap.

Besides the energy eigenvalues and the degeneracy, we need to know the explicit form of the ground-state wave function. Yang (15) did find the wave function for all the \((p, q)\) states; however, his solution is expressed explicitly that \( \Psi \), that is the product of the lowest \( SO(5) \) level, which is the lowest \( SO(5) \) level, and the single-particle energy spacing is finite. At \( v = 1 \), \( N \sim p^3 - R^6 \), the naively defined particle density \( N/R^4 \) would be infinite. However, we need to keep in mind that each particle also has an infinite number of isospin degrees because \( I \rightarrow \infty \). Taking this fact into account, we see that the volume of the configuration space, defined to be the product of the volume in orbital and isospin space, is \( R^3 \times R^3 \). Therefore, the density \( n = N/R^6 \) is actually finite in this limit. Using \( A = \{ n_1, n_2, n_3, n_4 \} \), the many-particle wave function is given by a Slater determinant

\[
\Phi(x_1, \ldots, x_N) = \Psi_{1}(x_1) \Psi_{2}(x_2) \Psi_{3}(x_3) \Psi_{4}(x_4)
\]

where each \( x_i \) is a \( \Psi \) and the isospin coordinate \( n_i \) is indexed by \( i = 1, 2, 3, 4 \). The effective inverse temperature is \( \frac{R}{\sqrt{mp}} \). The density correlation function \( \rho(x, x') = \frac{1}{(N-2)!} \int dx_3 \ldots dx_N \Phi(x, x', x_3, \ldots, x_N)^2 \) can be computed exactly and is given by

\[
\rho(x, x') = 1 - \left| \Psi_A(x) \Psi_A(x') \right|^2 \approx 1 - e^{-1/2 \left( -x_x + x'_x \right)^2}.
\]

where the explicit form of the single-particle wave function (Eq. 9) was used. In the approximation, we placed particle \( x \) on the north poles of both the orbital and the isospin space, i.e. \( x_x = 0 \), and expanded in terms of \( N_3 = R^2 (x_x^2 + x_y^2 + x_z^2) \) and \( N_2 = R^2 (x^2 + x^2 + x^2) \), and the limit \( n = \lim_{R \to \infty} \frac{R^2}{R^2} \). We see that just like in the QHE liquid, a particle is accompanied by a perfect correlation hole, gaussianly localized in its vicinity. The new feature in our case is that the incompressibility applies to both the charge and isospin channel.

Having discussed the generalization to the integer QHE, let us now turn to the fractional QHE. One can see that the many-body wave function \( \Phi_{\text{frac}} = \Phi^m(x_1, \ldots, x_N) \) with odd integer \( m \) is also a legitimate fermionic wave function in the lowest \( SO(5) \) level. This is so because the product of the basic spinors \( \Psi \) is always a legitimate state in the lowest \( SO(5) \) level. \( \Phi_{\text{frac}} \) is a homogeneous polynomial of \( \Psi(x) \) with degree \( p^m = mp \). Therefore, the degeneracy of the lowest \( SO(5) \) level in this case is

\[
d(mp, 0) = (p + 1)(p + m) \rightarrow \frac{1}{6} m^3 p^3 \text{, while the particle number is still} \quad N = d(p, 0).
\]

The filling factor in this case is \( \nu = N/mp, 0 \). Although \( \Phi_{\text{frac}} \) cannot be expressed in the Laughlin form of a single product, we can still use plasma analogy to understand its basic physics. \( \Psi_{\text{frac}} \) can also be interpreted as the Boltzmann weight for a classical fluid, whose effective inverse temperature is \( \beta_{\text{frac}} = \frac{\hbar^2}{m^2 R^4} \). As the correlation functions for the \( m = 1 \) case can be computed exactly, it is plausible that the \( m > 1 \) case has similar correlations; in particular, it is also an incompressible liquid. However, the effective parameters need to be rescaled properly in the fractional case. The effective magnetic length is given by \( \ell^2 = \frac{R}{\sqrt{mp}} = \frac{R}{\sqrt{mp}} \). This incompressible liquid supports fractionalized charge excitation with charge \( m \). Such a state may be described by a wave function of the form \( \Phi_{\text{frac}} \), where \( \Phi_{\text{frac}} \) is the wave function of the integer case, where one hole is removed from a given location in the bulk interior to the edge of the fluid. To our knowledge, this is the first time that a quantum liquid with fractional charge excitation has been identified in higher dimension \( d > 2 \).

Emergence of relativity at the edge. Before we go to the discussion of our model, let us first review how 1 + 1 dimensional relativity emerges at the edge of the 2D QHE problem. We shall restrict ourselves to the integer case only. In the III, there is no kinetic energy. The only energy is supplied by the confining potential \( V(r) \), which confines the particles in a circular droplet of size \( R \). Eigenfunctions in the III take the form \( \Phi_{\text{III}}(x) \)

\[
= x^2 \exp \left(-\frac{|z|^2}{4} \right).
\]

From this we see that a particle is localized in the radial direction at

\[
www.sciencemag.org SCIENCE VOL 294 26 OCTOBER 2001
825
\]
in this case gives rise to collective excitations of the spectrum. Mathematically, this effect manifests itself in terms of the different ways of combining the SO(4) angular momenta of a particle and a hole. Let us investigate the possibility of collective excitations in the spectrum. In a noninteracting Fermi system with the usual form of kinetic energy \( E = \mathbf{p}^2 / 2m \), a particle and a hole have a well-defined relative momentum but do not have a well-defined relative position, except in one spatial direction. Therefore, such a pair can only be “bound” through an attractive interaction. However, there are very special cases where the pair can be bound for kinematic reasons, without any interactions. In one dimension, the kinetic energy is approximately independent of the relative momentum; therefore, one can superpose states with different relative momenta to obtain a state with well-defined relative position. The resulting state is a bosonic collective mode. In our case, we find that the special nature of the wave function in the lowest SO(5) level leads to a similar form of the kinematic binding. Basically, there is no kinetic energy in the lowest SO(5) level, and a particle and a hole can be locked into a well-defined relative position without any kinetic energy cost. In our case, these collective excitations lie at the edge of the continuum states and are characterized by the total SO(4) quantum numbers \( (m_1, m_2) \), which is a positive integer and the \( \lambda = 0 \) case is counted only once. These states are formed by a macroscopic number of contractions of the spinor wave functions (Eq. 9) of a particle and a hole, and it can be shown explicitly that the wave function in the relative orbital and isospin coordinates are gaussianly localized. In this sense, a particle and a hole form a bound state and represent collective excitations of the system. In the flat space limit, the SO(4) symmetry group of \( S^3 \) reduces to the Euclidean group \( E_3 \) of the 3D flat space. The Euclidean group has two Casimir operators, and the magnitude of the momentum operator \( |\mathbf{p}| \) is determined by either \( T_1 \) or \( T_2 \), which in our case gives \( |\mathbf{p}| = nR \). As the energy is given by Eq. 12, the collective excitations have a relativistic linear dispersion relation \( E = \hbar \Omega - \Delta X \), with the speed of light given by \( c = \frac{\partial \Omega}{\partial X} \).
The second Casimir operator of the Euclidean group is the helicity, \( \lambda = J^\mu p_\mu \), where \( J \) is the total angular momentum of a particle. This quantity can be obtained from the SO(4) quantum numbers by \( \lambda = T_i - T_j \) \( (16) \). Therefore, the \( (T_1 = n/2, T_2 = T_3) \) state describes a relativistic spinless particle obeying the massless Klein-Gordon equation. The \( (T_1 = n/2, T_2 = T_3 + 1) \) and the \( (T_1 = T_2 + 1, T_2 = n/2) \) states describe massless photon states with left-handed and right-handed circular polarization. The associated fields satisfy Maxwell’s equation.

Hall current and noncommutative geometry. So far, we have discussed only the quantum eigenvalue problem. It is also instructive to discuss the classical Newtonian equation of motion derived from the Hamiltonian \( H = V(X) \), where \( H \) is given by Eq. 7. The classical degrees of freedom are the isospin vector \( I_i \), the position \( X_i \), and the angular momentum \( L_{ij} \); and their equations of motion can be derived from their Poisson bracket with the Hamiltonian. As we are interested in the equations of motion in the lowest \( SO(5) \) level, we can take the infinite mass limit \( M \rightarrow \infty \). In this limit, we obtain the following equations of motion

\[
\dot{X}_i = \frac{R^4}{\bar{T}} \frac{\delta V}{\delta X_i} F_{ab} J^b, \quad I_i = \epsilon_{ijk} a^j X_k J^j \quad (13)
\]

where the dot denotes the time derivative. Just as in the III problem, the momentum variables can be fully eliminated. However, the price one needs to pay for this elimination is that coordinates \( [X^a, X^b] \) become noncommuting. In fact, the projected Hamiltonian in the lowest \( SO(5) \) level is simply \( V(X) \). If we assume the commutation relation \( [X^a, X^b] = \delta^a_b \), then the orbital part of Eq. 13 can be derived from the Poisson bracket of \( X^a \) with \( V(X) \). If we expand around the north pole \( X_5 = R \), we finally obtain the following commutation relation

\[
[X^a, X^b] = 4i \hbar \eta_{ab} I_s \quad (14)
\]

This is the central equation underlying the algebraic structure of this work. It shows that there is a fundamental limit, \( \hbar_s \), for the measurability of the position of a particle.

The first equation in Eq. 13 determines the Hall current for a given spin direction \( J^\mu \) in terms of the gradient of the potential \( V(X) \), giving a direct generalization of the 2D Hall effect. From the second equation in Eq. 13, we see that the spin of a particle precesses around its orbital angular momentum (which becomes linear momentum in the flat space limit) with a definite sense.

**Conclusion.** At the conclusion of this work, we now know three different spatial dimensions where quantum disordered liquids exist: the 1D Luttinger liquid, the 2D quantum Hall liquid, and the 4D generalization found in this work. We can ask what makes these dimensions special. There is a special mathematical property that singles out these spatial dimensions. One, two, and four dimensional spaces have the unique mathematical property that boundaries of these spaces are isomorphic to mathematical groups, namely the groups \( \mathbb{Z}_2 \), \( U(1) \) and \( SU(2) \). No other spaces have this property. It is the deep connection between the algebra and the geometry that makes the construction of nontrivial quantum ground states possible. Other related mathematical connections are reviewed and summarized in (11). The 4D generalization of the QHE offers an ideal theoretical laboratory to study the interplay between quantum correlations and dimensionality in strongly correlated systems. It would be interesting to study our quantum wave functions on 4D manifolds with nontrivial topology and investigate whether different topologies of four manifolds correspond to degeneracies of our many-body ground states. The quantum plateau transition in the 2D QHE is still an unsolved problem; one could naturally ask if the plateau transition in four dimensions can be understood better because of the higher dimensionality. In 2D QHE, quasi-particles have both anyonic and exclusion statistics. The former cannot exist in four dimensions; the question is whether quasi-particles in our theory would obey exclusion statistics in the sense of Hal-dane. To address these questions, it is important to construct a field theory description of the 4D quantum Hall liquid, in analogy with the Chern-Simons-Landau-Ginzburg theory of the QHE.

In this work, we investigated the possibility of modeling relativistic elementary particles as collective boundary excitations of the 4D quantum Hall liquid. Similar connections between condensed-matter and particle physics have been explored before (20 -24). There are important aspects unique to the current problem (25). The single-particle states are hugely degenerate, which enables the limit of zero inertia mass \( M \rightarrow 0 \) and completely removes the nonrelativistic dispersion effects. This limit is hard to take in usual condensed-matter systems. The single-particle states also have a strong gauge coupling between iso-spin and orbital degrees of freedom, which is ultimately responsible for the emergence of the relativistic helicity of the collective modes. This type of coupling is not present in usual condensed-matter systems. The vanishing of the kinetic energy is the lowest \( SO(5) \) levels enables binding of a particle and a hole into a pointlike collective mode. The most remarkable mathematical structure is the noncommutative geometry (Eq. 14), which expresses a \( SU(2) \) co-cycle structure of the magnetic translation. Although progress reported in this work is still very...
limited, we hope that this framework can stimulate investigations on the deep connection between condensed-matter and elementary particle physics.

References and Notes

RESEARCH ARTICLES

Groundwork for a Rational Synthesis of \( \text{C}_{60} \):
Cyclodehydrogenation of a \( \text{C}_{60} \text{H}_{30} \) Polyarene

Margaret M. Boorum, Yury V. Vasiliev, Thomas Drewello,* Lawrence T. Scott* 1

A \( \text{C}_{60} \text{H}_{30} \) polycyclic aromatic hydrocarbon (PAH) that incorporates all 60 carbon atoms and 75 of the 90 carbon-carbon bonds required to form the fullerene \( \text{C}_{60} \) has been synthesized in nine steps by conventional laboratory methods. Laser irradiation of this \( \text{C}_{60} \text{H}_{30} \) PAH at 337 nanometers induces hydrogen loss and the formation of \( \text{C}_{60} \) as detected by mass spectrometry. A specifically labeled \( \text{[C}_{60} \)\text{H}_{30} \) retains all three \( ^{13} \text{C} \) atoms during the cage formation process. A structurally related \( \text{C}_{46} \text{H}_{24} \) PAH that lacks the three peripheral benzene rings cannot be transformed into \( \text{C}_{60} \) whereas the next higher homolog, a \( \text{C}_{40} \) with \( \text{C}_{60} \text{H}_{30} \) PAH, which then loses hydrogen to give \( \text{C}_{60} \text{H}_{30} \). These control experiments verify that the \( \text{C}_{60} \) is formed by a molecular transformation directly from the \( \text{C}_{46} \text{H}_{24} \) PAH and not by fragmentation and recombination in the gas phase.

Despite more than a decade of intensive research on fullerenes (1), chemists worldwide still have no general methods or strategies available for the rational synthesis of these polyhedral carbon allotropes as discrete, preselected targets. Under carefully controlled conditions, the vaporization of graphite generates substantial amounts of \( \text{C}_{60} \) and \( \text{C}_{70} \); however, this complicated process remains poorly understood and is intolerant to alteration (2). Higher fullerenes can be obtained from this source only in minuscule amounts through tedious chromatographic separations (3) and likely will never be available in quantity except by rational synthesis.

Before we can hope to develop rational syntheses of individual higher fullerenes, the goal of synthesizing \( \text{C}_{60} \) by rational methods must first be met. In this connection, the research groups of Diederich and colleagues (4), Rubini et al. (5), and Tobe et al. (6–8) have all prepared macromolecular polyalkynes that shed multiple appendages when subjected to laser desorption/ionization (LDI), and the high-energy intermediates thus generated collapse to \( \text{C}_{60} \) in a mass spectrometer. The considerable ambiguity about which atoms in these molecular precursors become bonded to which other atoms as the fullerene takes shape, however, precludes characterization of these processes as entirely “rational” syntheses. Prinzbach et al. (9) recently reported a genuinely rational synthesis of icosahedral \( [5] \text{fullerene-C}_{60} \) in which substituents were removed from a preformed dodecahedrane cage, but extensions of this approach to syntheses of fullerenes comprising 60 or more carbon atoms are likely to be difficult. In LDI experiments, the reactive \( \text{C}_{20} \) entities prepared in this way can be made to oligomerize and fuse into \( \text{C}_{60} \) (10).

Here, we report the synthesis of a stable polymeric aromatic hydrocarbon (PAH) that incorporates all 60 of the carbon atoms and 75 of the 90 carbon-carbon bonds required to form \( \text{C}_{60} \) and its laser-induced cyclodehydrogenation to \( \text{C}_{60} \) (Fig. 1). Control experiments establish that the \( \text{C}_{60} \) formed in the final step comes from a direct molecular transformation (or “zipping up”) of the synthetic PAH (6), as suggested in Fig. 1, and not by a laser-induced degradation of the hydrocarbon to smaller fragments that recombine in a thermodynamically driven manner, as in the laser-induced vaporization of graphite (2).

Several research groups have independently conceived of a laboratory synthesis of \( \text{C}_{60} \) from 6 or structurally related 60-carbon compounds. Wang and Shevlin in Alabama were the first to report preliminary experimental work in this direction (11). Their approach can be traced back to an earlier proposal from the same laboratory (12); however, they encountered difficulty in assembling the 60-carbon fullerene precursor. Our synthesis of 6 was designed to ensure a regio-regular head-to-tail cyclotrimerization of a splay-shaped 20-carbon precursor, 5. While our work was under way, Sarobe et al. in the Netherlands found that attempts to synthesize 6 from a different precursor without controlling the head-to-tail regiochemistry gave the \( \text{C}_{5} \)-symmetric PAH 6 only as a minor component in an inseparable mixture of \( \text{C}_{50} \text{H}_{30} \) regioisomers (13). Gomez-Lor et al. in Spain later prepared 6 as a single regioisomer by threefold annulation of truxene, a venerable 27-carbon, \( \text{C}_{5} \)-symmetric PAH (14, 15). Neither Sarobe et al. nor Gomez-Lor et al., however, were able to convert their synthetic \( \text{C}_{50} \text{H}_{30} \) material to \( \text{C}_{60} \).

Our synthesis of 6 (16) began with commercially available (1-bromoethyl)benzene, 1, and 2-azaphthaldehyde (17). These were joined by a Wittig reaction to give alkenes 2 as a mixture of \( \text{E} \)- and \( \text{Z} \)-isomers in a combined yield of 79% after purification. Oxidative photocyclization of 2 under standard conditions

1. Department of Chemistry, Merkert Chemistry Center, Boston College, Chestnut Hill, MA 02467, USA.
2. Department of Chemistry, University of Warwick, Coventry CV4 7AL, UK.
3. To whom correspondence should be addressed. E-mail: t.drewello@warwick.ac.uk, lawrence.scott@bc.edu

25. Detailed calculations appear as supplementary Web material on Science Online at www.sciencemag.org/cgi/content/full/294/5543/823/DC1.